B.Sc Physics (Hons) Part I

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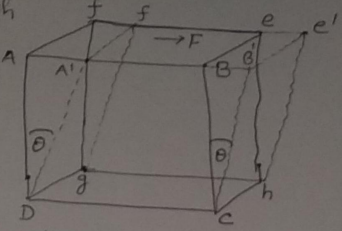
Ques If Y, K and σ be the young's modulus, Bulk modulus and Poisson's ratio. Then Prove that $K = \frac{Y}{3(1-2\sigma)}$

Ans Let a unit cube ABCDefgh and forces T_x, T_y and T_z acts ⊥ to the faces BehC and AfgD, efgh & ABCD, ABef and CDgh respectively.



If α is the increase in length per unit length per unit tension along the direction of the force, then the elongation produced in the edges AB, Be, BC will be $T_x\alpha$, $T_y\alpha$ and $T_z\alpha$ respectively.

If β is the contraction produced per unit length per unit tension in the direction ⊥ of the force, then the Contraction produced ⊥ to the edges AB, Be & BC will be $T_x\beta$, $T_y\beta$ and $T_z\beta$. respectively.

Thus the lengths of the edges are as follows

$AB = 1 + T_x\alpha - T_y\beta - T_z\beta$

$Be = 1 + T_y\alpha - T_x\beta - T_z\beta$

$BC = 1 + T_z\alpha - T_x\beta - T_y\beta$

Hence the new volume of the cube becomes

$$AB \times Be \times BC = (1 + T_x\alpha - T_y\beta - T_z\beta) \times (1 + T_y\alpha - T_x\beta - T_z\beta) \times (1 - T_z\alpha - T_x\beta - T_y\beta)$$

Since α and β are very small quantities terms containing their squares and higher power can be neglected

$$\therefore \text{Volume} = 1 + \alpha(T_x + T_y + T_z) - 2\beta(T_x + T_y + T_z)$$

$$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z)$$

If the deforming forces acting on the three faces are equal

Then $T_x = T_y = T_z = T$

$$\therefore \text{Volume} = 1 + 3T(\alpha - 2\beta)$$

If instead of applying a stretching force T outwardly, a pressure P is applied on all the faces to compress the cube, the contraction in volume

$= 3P(\alpha - 2\beta)$

$\therefore$ Volumetric Strain $= 3P(\alpha - 2\beta)/1$ ($\because$ It is unit cube

Hence Bulk modulus $K = \dfrac{\text{Stress}}{\text{Volumetric Strain}}$

$= \dfrac{P}{3P(\alpha - 2\beta)}$

$= \dfrac{1}{3(\alpha - 2\beta)}$ . . . . . (1)

$\therefore K = \dfrac{\frac{1}{\alpha}}{3(\alpha - 2\beta)/\alpha} = \dfrac{\gamma}{3(1 - 2\sigma)}$

$\because \frac{1}{\alpha} = \gamma$ and $\frac{\beta}{\alpha} = \sigma$ (Poisson's ratio)